Spectral calibration of exponential Lévy models.

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Workshop on Financial Modeling with Jump processes
Ecole Polytechnique (France), 6–8 September 2006.

Abstract

The exponential Lévy model reflects the assumption that the log returns of the asset evolve independently and with identical distribution for the same time steps, which is plausible for liquid markets and not too long time horizons. This basic model class, first introduced by [?], has been considered recently for a variety of pricing and optimisation problems in finance, cf. the recent works by [?], [?], [?] and the references therein.

The work on calibration methods for financial models based on Lévy processes has mainly focused on certain parametrisations of the underlying Lévy process with the notable exception of [?]. Since the characteristic triplet of a Lévy process is a priori an infinite-dimensional object, the parametric approach is always exposed to the problem of misspecification, in particular when there is no inherent economic foundation of the parameters and they are only used to generate different shapes of possible jump distributions. The goal of this work is to investigate mathematically the problem of nonparametric inference for the Lévy triplet when the asset price (S_t) follows an exponential Lévy model

$$S_t = Se^{rt+X_t} \text{ with a Lévy process } X_t \text{ for } t \geq 0. \tag{1}$$

We suppose that at time $t = 0$ we dispose of prices for vanilla European call and put options on this asset with different strike prices and possibly different maturities. By basing our estimation on option data we draw inference on the underlying risk neutral price process, which in general cannot be determined from historical price data due to the incompleteness of the Lévy market.

The observed option prices will be slightly unprecise due to bid-ask spreads or other market frictions. In the ideal case of precise observations for all possible strike prices the state price density and hence the Lévy triplet can be uniquely identified using the formula by [**?**]. Under the realistic model of finitely many noisy observations we cannot hope to determine the triplet correctly, we should rather try to provide an estimator which is as good as possible for the given accuracy of the data. This optimality property is usually assessed by the minimax paradigm, which measures the inherent complexity of the statistical problem class. One of the our main results is a lower bound, showing that already in the simple exponential Lévy model the estimation problem is in general severely ill-posed, that is, the estimation error for any part of the Lévy triplet as a function of the accuracy of the observations will only converge with a logarithmic rate for any conceivable estimation procedure.

On the other hand, we propose an explicit construction of an estimator that attains this optimal minimax rate. The procedure is based on the inversion of the explicit pricing formula via Fourier transforms by [**?**] and a regularisation in the spectral domain. Using the Fast Fourier Transformation, the procedure is easy to implement and yields good results in simulations in view of the severe ill-posedness (see [**?**] and [**?**]). In comparison with standard statistical ill-posed problems, the main challenges are the nonlinearity involved and the complex interplay between the jump measure as nonparametric part and the drift and diffusion coefficient as parametric parts.

Our procedure can be adapted to different models as long as the inverse transformation from the option prices to the characteristic function can be calculated and the unknown quantities can be determined from the structure of the characteristic function. Time changed Lévy models and affine jump diffusion models will be considered and analyzed.

Keywords: European option, jump diffusion, minimax rates, severely ill-posed, nonlinear inverse problem, spectral cut-off, time changed Lévy models.

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