A Lévy-driven structural model for the valuation of CDOs and other credit derivatives

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1 Introduction and Motivation

In the market for Collateralized Debt Obligations (CDOs), the Gaussian copula model has emerged as the standard tool for valuation purposes. However, it provides a rather poor fit to actually observed prices of CDO-tranches, which is why a large literature suggests extensions and alternatives (see e.g. Burtschell, Gregory and Laurent (2005)). We contribute to this literature by introducing a structural modelling approach that is backed by a sound economic and empirical rationale: We model the firms’ asset value processes as exponential Lévy-processes, which allows us to incorporate features such as skewness and heavy-tailedness of asset return-distributions. Furthermore, the Lévy-approach enables us to move beyond the concept of linear correlation and to incorporate a notion of dependence that is particularly suited for modelling joint default behaviour, but absent in Gaussian models, namely tail dependence. As our model is of dynamic nature, it offers the benefit of describing credit spreads, correlation smiles and (joint) default behaviour dynamically. For this reason, and in contrast to the standard Gaussian copula model, it provides a coherent framework for pricing and hedging all types of credit derivatives, thus helping to avoid the inconsistent practice of using different models for different types of credit derivatives, though possibly written on the same underlyings.

2 The basic setup

This section describes the basic Black-Cox-type structural model. Let $n$ be the number of firms under consideration, and let $A_j$, $1 \leq j \leq n$, be the asset value process of the $j$th firm. The modelling horizon is denoted by $T$. We posit that $A_j$ follows an exponential Lévy process under a martingale measure $Q$,

$$A_j(t) = A_j(0) \exp \left( L_j(t) + \int_0^t (r(s) - q_j(s)) \, ds - \log(\phi_j(\bar{i})) t \right),$$

where $L_j$ is a Lévy-processes and $\phi_j(u) = E \left( e^{iuL_j(1)} \right)$ is the characteristic function of $L_j(1)$. $(r(t))$ and $(q_j(t))$ denote the processes of the continuously compounded riskless interest rate and the continuous payout rate, respectively, which for simplicity are both assumed to be deterministic. We assume that firm $j$ defaults as soon as its asset value drops below a firm-specific compounded deterministic (but possibly time-dependent) default-barrier $B_j(t)$ on one of the observation-dates $0 = t_0 < t_1 < \cdots < t_m = T$. We suggest several parametrizations for the default barrier.

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Emulating the factor approach taken in standard copula models, we introduce correlation between the individual asset value processes by assuming that all asset values are driven by one or more common factors $M_i$ (which could represent macro-economic conditions or the economic situation of an industry sector etc.) and an idiosyncratic factor $N_j$:

$$L_j(t) = \sum_{i=1}^{l} \rho_{ji} M_i(t) + \sqrt{1 - \sum_{i=1}^{l} \rho_{ji}^2} N_j(t),$$

with $\sum_{i=1}^{l} \rho_{ji}^2 \leq 1$. All processes $M_i$ and $N_j$ are assumed to be independent Lévy processes, so that $L_j$ is again a Lévy process. While our considerations hold for general Lévy processes, we now narrow our focus to the particularly tractable subclass of Normal Inverse Gaussian (NIG) processes that has been successfully used in describing equity dynamics. We posit a one-factor model of the form

$$L_j(t) = \rho M(t) + \sqrt{1 - \rho^2} N_j(t),$$

with $0 \leq \rho \leq 1$ a constant, and the common factor $M$ and the idiosyncratic factors $N_j$ independent NIG-processes. Using that the NIG-class is stable both under convolution and scaling, we suggest a particularly parsimonious parametrization which ensures that the $L_j$ are again NIG-processes.

### 3 Numerical issues, model properties and empirical performance

Monte Carlo simulation of a credit portfolio poses considerable computational challenges. Often, not only the number of names in the portfolio is fairly large (for example the baskets underlying the CDX or the iTraxx indices consist of 125 names), but also the time period that has to be covered by the simulation is considerable – CDOs with times to maturity of 5 or 10 years are common. To enhance the numerical tractability of our model, we propose a variance reduction technique, which builds on the idea of choosing the default barriers in dependence of a set of simulated paths, as opposed to a priori. We demonstrate the effectiveness of this approach by means of a numerical example. Subsequently, we calibrate both the Gaussian and NIG structural models to the market prices of a set of CDO tranches. Comparing the calibration results, we find that prices of CDOs can be almost perfectly matched with the NIG-model, while the Gaussian model proposed by Hull, Predescu and White (2005) produces significantly higher calibration errors. We then investigate the properties of the NIG-model by studying the typical shapes of Credit Default Swap spreads and portfolio loss distributions, and the typical patterns of default correlation and tail dependence it gives rise to, and contrast them with the respective properties of the Gaussian structural model. Comparing the results, we observe significant differences between these model classes. We conclude with a discussion of model risk.

### 4 Summary

In sum, we propose a flexible structural model for the valuation of credit derivatives that combines a sound economic, dynamic approach with comprehensiveness, numerical tractability, appealing theoretical and empirical features, and superior pricing performance.

### 5 References
