Mean-variance hedging in Lévy models with stochastic volatility

Terence Chan∗, Jozef Kollar†, Anke Wiese∗
MACS, Heriot-Watt University, Edinburgh, UK

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Abstract

We consider an incomplete market where stock price fluctuations are modelled by a geometric Lévy process with stochastic volatility, modelled as an exogenous factor. Such a market is incomplete and there are infinitely many equivalent martingale measures. We study the mean-variance hedging problem that requires the calculation of the variance-optimal martingale measure. The mean-variance hedging approach minimizes the expectation of the square difference between the value of the strategy and the underlying contingent claim at the maturity, among all self-financing strategies. The variance optimal martingale measure, as its name suggests, has minimal variance among all martingale measures. We stress that model setup, variance and expectation above are defined and calculated under the “real-world” measure.

The mean-variance hedging problem has been studied extensively in the case when the price process is a continuous semimartingale, see for instance [5] and [8]. Without some very restrictive assumptions on the model, there are only few results in the discontinuous case, see for instance [1] and [3].

We follow the first approach, in which the calculation of the mean variance strategy consists of three main steps: 1) determining the variance optimal martingale measure; 2) finding the evolution of the model under the variance optimal martingale measure; 3) finding the Galtchouk-Kunita-Watanabe decomposition of the contingent claim relative to a certain numéraire under the measure relating the variance optimal martingale measure and this numéraire.

We will determine the variance-optimal martingale measure explicitly. To be able to calculate the second step, we assume Markovian framework. We then set up the model parameters such that the drift of the volatility process under the variance-optimal martingale measure is related to the conditional Laplace transform of the integrated variance process.

*research advisor
†2nd year PhD student
class of processes we can explicitly derive the solution to the mean-variance hedging problem. Working in the context of Lévy processes, the natural candidate for the stochastic volatility process is then the class of non-Gaussian positive Ornstein-Uhlenbeck type processes, which we consider in the numerical example section.

Only the last step is different in the calculation of the mean variance hedging strategy when the continuous and discontinuous cases are compared. We will observe in our example this difference and show why the decomposition from third step has to be used in the discontinuous setting as soon as some very strong assumptions are relaxed. Because of the Markovian structure, finding the Galtchouk-Kunita-Watanabe decomposition reduces to a PIDE problem.

Note: The section on numerical examples is not finished yet.

Keywords: Mean-variance hedging, variance-optimal martingale measure, Lévy processes, stochastic volatility, Ornstein-Uhlenbeck processes

References


