Lévy copulas: dynamics and transforms of Upsilon-type

Ole E. Barndorff-Nielsen
Department of Mathematical Sciences
University of Aarhus

Alexander M. Lindner (speaker)
Centre for Mathematical Sciences
Munich University of Technology

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Abstract

The concept of copulas for multivariate probability distributions (or distributional copulas, for short) has an analogue for multivariate Lévy measures, called *Lévy copulas*. The latter concept was introduced in a paper by Tankov [6] for Lévy measures on $\mathbb{R}_+^m$, and extended to Lévy measures on $\mathbb{R}^m$ by Kallsen and Tankov [5], see also the book by Cont and Tankov [4]. Similar to copulas, a Lévy copula describes the dependence structure of a multivariate Lévy measure. The Lévy measure is then completely characterised by knowledge of the Lévy copula and the margins.

In this talk we shall discuss several aspects of the Lévy copula concept, where we shall restrict attention to Lévy measures having support on $\mathbb{R}_+^m$. We first establish a limit result for sequences of Lévy measures and Lévy copulas: a sequence of Lévy measures converges vaguely to another Lévy measure if and only if the marginal Lévy measures converge vaguely, and the associated Lévy copulas converge pointwise on a suitable subset of $[0,\infty]^m$.

We then show that every Lévy copula C defines itself a Lévy measure ν_C with normalised 1-stable margins $d(\nu_C)_i(x_i) = x_i^{-2}\,dx_i$, in a canonical way. This result is then used to characterise all homogeneuous Lévy copulas, i.e. Lévy copulas C which satisfy

$$C(tx) = t\,C(x) \quad \forall\, x \in [0,\infty]^m \quad \forall\, t > 0.$$

Finally, we pay attention to the construction of Lévy measures and distributions with special structures and prescribed margins. Suppose that $\nu_1, \ldots, \nu_m$ are one-dimensional Lévy measures, all of which have a similar structure, such as being selfdecomposable, say. We outline a general

scheme how Lévy copulas can be used to construct a Lévy measure ν with margins $\nu_1, \ldots, \nu_m$ and which has the same structure, e.g. selfdecomposability. Apart from Lévy copulas the method requires certain mappings which are of *Upsilon type*, which were introduced by Barndorff-Nielsen and Thorbjørnsen [3] and Barndorff-Nielsen, Maejima and Sato [2].

The talk is based on [1].

Keywords: Homogeneous Lévy copula, limits of Lévy copulas, selfdecomposable distribution.

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