Abstract

A model for a set of stock prices is said to be convexity preserving if the price of any convex European claim is convex as a function of the underlying stock prices at all times prior to maturity. As is well-known, this property is intimately connected to certain monotonicity properties of the option price with respect to volatility and other parameters of the model. Generally speaking, if the option price is convex at all fixed times, then it is also increasing in the volatility. This robustness property motivates the study of convexity preserving models in finance.

Although these issues have been studied quite extensively during the last decade, compare [?], [?], [?], [?] and [?] for the case of one-dimensional diffusion models, [?] and [?] for several-dimensional diffusion models, [?] for one-dimensional jump-diffusion models and [?] for exponential semimartingale models, the general picture for more advanced models is not yet fully understood. In [?], a sufficient condition for the preservation of convexity in one-dimensional models with jumps is provided. That condition, however, is not a necessary condition for preservation of convexity. The main contribution of the present paper is to give a necessary condition for convexity to be preserved in jump-diffusion models in arbitrary dimensions. We also use this necessary condition to show that, within a large class of possible models, the only higher-dimensional convexity preserving models are the ones with linear coefficients.

To analyze the convexity of an option price we employ the characterization of the price as the unique viscosity solution to a parabolic integro-differential equation

\[ \frac{\partial u}{\partial t} = \mathcal{A}u + \mathcal{B}u \]

with an appropriate terminal condition. In this equation, \( \mathcal{A} \) is an elliptic differential operator associated with the continuous fluctuations of the stock price processes, whereas \( \mathcal{B} \) is an integro-differential operator associated with the jumps of the stock price processes. Preservation of convexity of the solution to the equation (??) is dealt with using the notion of locally convexity.

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preserving (LCP) operators. This concept was introduced and analyzed in [?], and also used in [?] and [?]. Following these references, we show that the condition that \( M = A + B \) is LCP at all points is necessary for convexity to be preserved. We also show that \( M \) is LCP if and only if both \( A \) and \( B \) are LCP, i.e.

\[
\begin{align*}
\{ \text{The model is convexity preserving} \} & \quad \implies \quad \{ \text{\( M \) is LCP at all points} \} \\
\{ \text{Both \( A \) and \( B \) are LCP} \} & \quad \iff \quad \{ \text{\( A \) and \( B \) are LCP at all points} \}.
\end{align*}
\]

Thus the characterization of LCP models breaks down into two easier problems: (i) to describe which diffusion models are LCP, and (ii) to describe which jump structures are LCP. Issue (i) has been dealt with in [?] and [?], and (ii) is dealt with in the present article.

The present paper is organized as follows. First we introduce the model and we motivate the study of convexity preserving models by means of a monotonicity result. We then prove a technical regularity result which is used in the sequel. Thereafter, we introduce the LCP-condition, and we show that a model is convexity preserving only if both the differential operator \( A \) and the integro-differential operator \( B \) are LCP at all points. Then we investigate which jump structures are LCP. This investigation is continued for models with only a finite number of possible jump sizes, where we show that, within a large class of possible higher-dimensional models, all convexity preserving models have linear diffusion coefficients and jump structures.

References


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