

A market model for forward variance and VIX futures

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Abstract

The S&P500 options market extends beyond the implied volatility surface: the VIX index tracks in real time the 30 days at-the-money implied volatility, and today VIX futures and options trade liquidly, hence implying a mean value and volatility surface for the 30 days at-the-money implied volatility observed at future dates. Usual stochastic volatility models can not be calibrated to be consistent with both the implied volatility surface and this additional data, and can neither reproduce realistic forward variance and VIX futures curves shapes. Markovian models therefore have to get higher dimensional and calibration becomes hard. The alternative approach we introduce is to build a joint market model for the underlying, its forward variance and its VIX futures, based on Bergomi's joint diffusion of the underlying and its forward variance. Any initial forward variance and VIX futures curves are therefore matched by construction, and the parameters can be instantaneously calibrated to at-the-money skews, at-the-money 30 days forward skews, and VIX options implied volatility. Our setup offers a natural framework to hedge volatility exposure with VIX futures and volatility of volatility exposure with VIX options.