Pros and Cons of the Finite Element Methods for Option Pricing

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Abstract

The finite element method is particularly well suited to the numerical solution of the partial differential equation of finance for two reasons: mesh adaptivity and a posteriori error estimates; however it can be slow. The main applications of the method will be recalled here and these advantages illustrated; a computation mixing Monte-Carlo and Partial Differential equation for a basket option will also be presented.

Ito Calculus applied to the Stochastic definition of option pricing models yields partial differential equations or integro-differential partial differential equations for which three or four classes of numerical methods can be used.

Finite Difference Methods are by far the simplest, except when mesh adaptivity is required in which case it is rather difficult to control the numerical error. *Finite Volume Methods* are not really natural, except in the case of Asian options and so are the *Spectral Methods* because the volatilities are not constant in the interesting cases

Finite Element Methods seem at first unnecessarily complex for finance where a large class of problems are one dimensional in space and yet rather easy to implement in practice as we shall see.

The Finite Element Method has been invented for solid mechanic engineering around 1950; it is an extension of the theorem of virtual work. Its generalization to other problems has been done by applied mathematicians mostly through the concept of variational formulations and weak forms of the partial differential equations. Consequently it is indifferent to uniform or arbitrary mesh and since a posteriori error indicators are available it is possible to tune the mesh accordingly; this property is very useful to compute accurately the exercise region of an American option or a multidimensional basket option.

Large drift terms can be handled by the Galerkin-Characteristic method, a technicality which is not always known, but essential for Asian option.

Finally the curse of dimension can be dealt with in a number of ways, from sparse grids to mixed Monte-Carlo/PDE methods.