

Measuring and pricing gap risk: from CPPI funds to gap options

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Abstract

In this talk we give examples of options and portfolio management strategies which are particularly sensitive to jump risk, and provide explicit formulae for their pricing in risk management, in models based on Levy processes.

In the first part of the talk we concentrate on the constant proportion portfolio insurance (CPPI) strategy. This strategy allows an investor to limit downside risk while retaining some upside potential by maintaining an exposure to risky assets equal to a constant multiple of the cushion, the difference between the current portfolio value and the guaranteed amount. Whereas in diffusion models with continuous trading, this strategy has no downside risk, in real markets the portfolio value may go below the guaranteed level in case of a sudden downside move (gap) of the underlying. We study the behavior of CPPI strategies in models where the price of the underlying portfolio may experience downward jumps. Our framework leads to analytically tractable expressions for the probability of hitting the floor, the expected loss and the distribution of losses. This allows to measure the gap risk but also leads to a criterion for adjusting the multiplier based on the investor's risk aversion.

In the second part of the talk, we focus on, gap notes, a new class of exotic options designed by some banks to sell off the risk of rapid downside moves in the price of the underlying, a risk to which CPPI strategies are particularly sensitive. We show that to price and manage gap notes, jumps must necessarily be included into the model, and present explicit models and pricing formulas in the single-name and multi-name case.